

## TOR AND TORSION ON A COMPLETE INTERSECTION

ABSTRACT. Let  $(R, \mathfrak{m})$  be a complete intersection, that is, a ring whose  $\mathfrak{m}$ -adic completion is the quotient of a regular local ring by a regular sequence. Suppose  $M$  and  $N$  are finitely generated  $R$ -modules. We give a necessary condition for the vanishing of  $\mathrm{Tor}_i^R(M, N)$  for all  $i \gg 0$  in terms of the intersection of certain affine algebraic sets associated to  $M$  and  $N$ . We apply this condition to the study of torsion in tensor products. For example, we show that if  $R$  is a domain and  $M$  is an  $R$ -module of infinite projective dimension then there exist infinitely many  $n$  for which the tensor product of  $M$  with one of its  $n^{\mathrm{th}}$  syzygy modules has torsion.

We also give a sufficient condition for the vanishing of  $\mathrm{Tor}_i^R(M, N)$  for all  $i \gg 0$  in terms of the ability to lift  $M$  and  $N$  to “disjoint” complete intersections of smaller codimension. We use this condition to construct tensor products of non-free modules which are maximal Cohen-Macaulay.