EXISTENCE OF UNLIFTABLE MODULES

ABSTRACT. Let $(Q, \mathfrak{n})$ be a commutative Noetherian local ring, and let $R = Q/(x)$ where x is a non-zerodivisor of Q contained in $\mathfrak{n}$. Then a finitely generated R-module M is said to lift to Q if there exists a finitely generated Q-module M' such that x is M'-regular and $M \cong M'/xM'$. The lifting theorem of Buchsbaum and Eisenbud — which is a consequence of their structure theorem for finite free resolutions — states that if a cyclic R-module $R/\mathfrak{a}$ has projective dimension 3 and $\mathfrak{a}$ is generated by 3 elements, then $R/\mathfrak{a}$ lifts to Q. In this paper, we give a general construction of finitely generated R-modules of finite projective dimension over R which fail to lift to Q provided $x \in \mathfrak{n}^2$ and the depth of R is at least 2. In fact, our technique produces examples which show that the lifting theorem of Buchsbaum and Eisenbud cannot in general be extended to cyclic R-modules of projective dimension greater than 3, nor to projective dimension 3 cyclic modules $R/\mathfrak{a}$ where $\mathfrak{a}$ needs more than 3 generators.