COMPLEXITY AND TOR ON A COMPLETE INTERSECTION

ABSTRACT. Let $(R, \mathfrak{m})$ be a complete intersection, that is, a local ring whose $\mathfrak{m}$-adic completion is the quotient of a regular local ring by a regular sequence. Suppose M is a finitely generated R-module. It is known that the even and odd Betti sequences of M are eventually given by polynomials of the same degree n; the complexity of M is the non-negative integer $n+1$. We use this notion of complexity to study the vanishing of $\operatorname{Tor}_i^R(M, N)$ for finitely generated modules M and N over a complete intersection R. We prove several theorems dealing with rigidity of Tor, which are generalizations and, in certain situations, improvements of known results. The main idea of these rigidity theorems is that the number of consecutive vanishing Tors required in the hypothesis of a rigidity theorem depends more on the minimum of the complexities of M and N rather than on the codimension of R. We give examples showing that this dependence is sharp. We also show that if $M \otimes_R N$ has finite length, then for sufficiently high indices two consecutive vanishing Tors force the vanishing of all higher Tors.

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